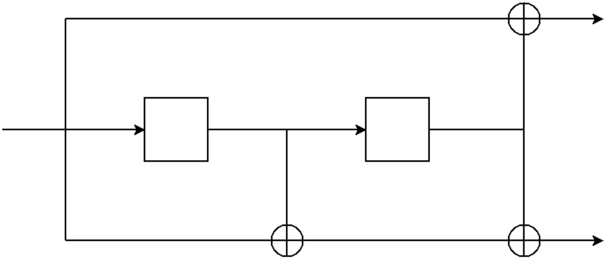
Exemple

Code de convolution $(n, k, m) = (2, 1, 2)$ de polynômes générateurs

$$\mathbf{g}_1(X) = 1 + X^2 \quad \text{et} \quad \mathbf{g}_2(X) = 1 + X + X^2.$$



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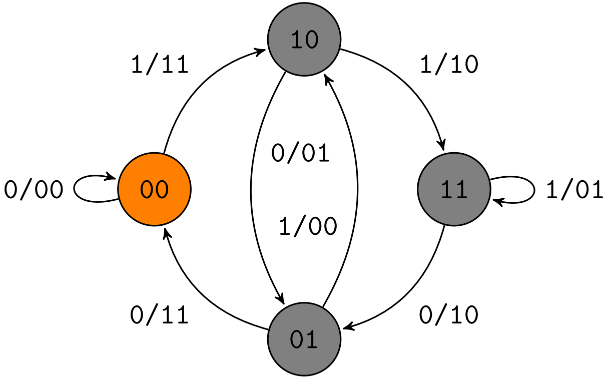
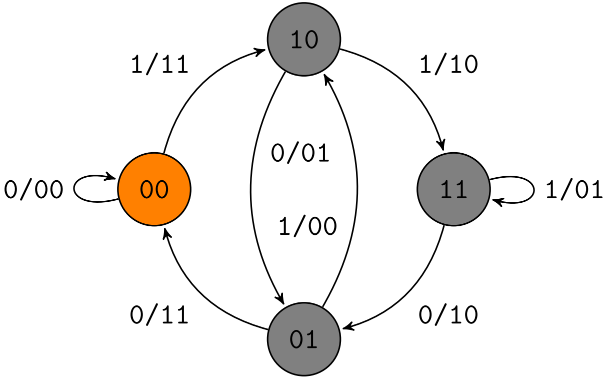


Encodage

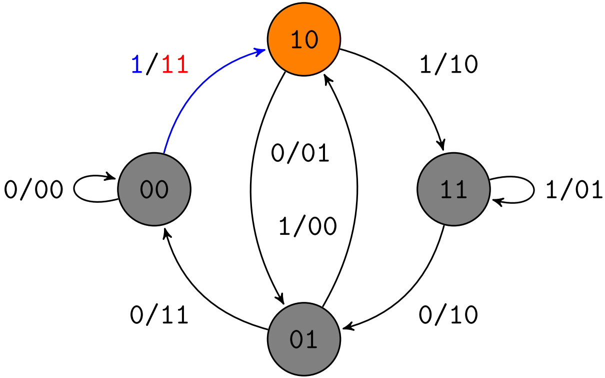
$$\mathbf{u} = 11001$$
$$\mathbf{v} = ?$$



Encodage

$\mathbf{u} = 11001$

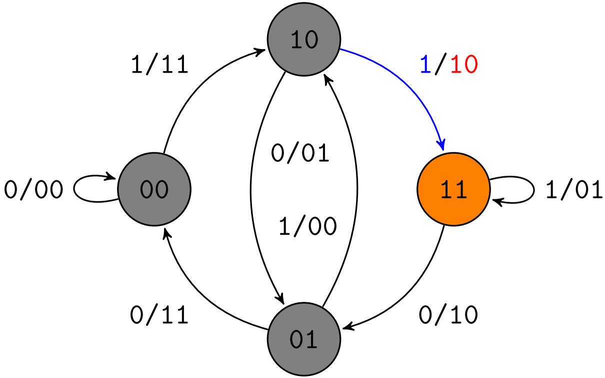
$\mathbf{v} = 11$



Encodage

$\mathbf{u} = 11001$

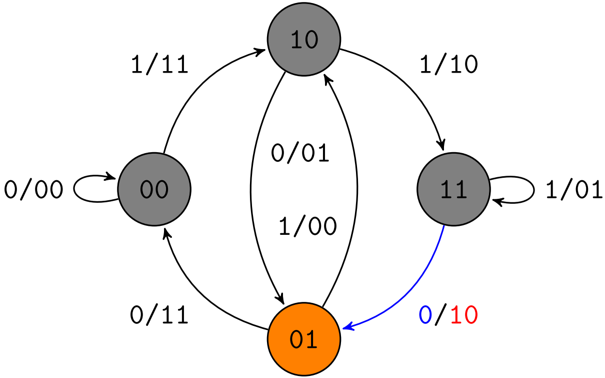
$\mathbf{v} = 1110$



Encodage

$$\mathbf{u} = 11\textcolor{blue}{0}01$$

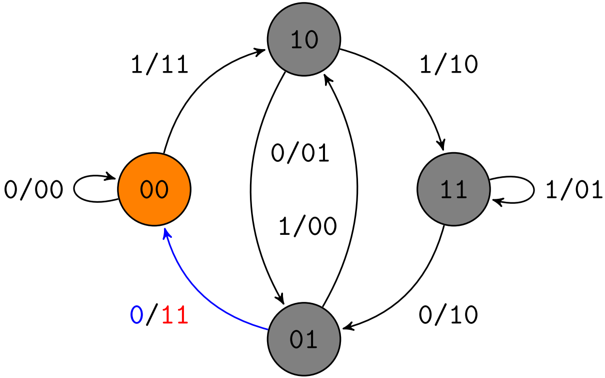
$$\mathbf{v} = 1110\textcolor{red}{10}$$



Encodage

$$\mathbf{u} = 11001$$

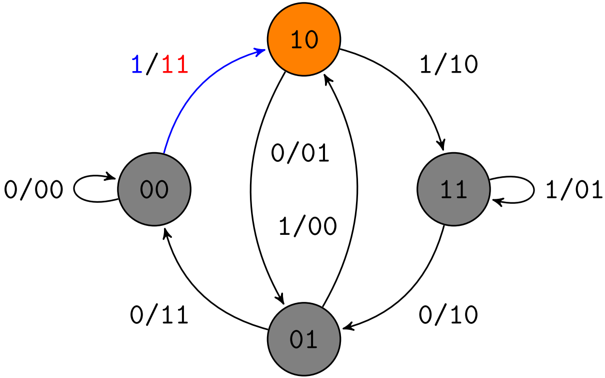
$$\mathbf{v} = 11101011$$



Encodage

$$\mathbf{u} = 11001$$

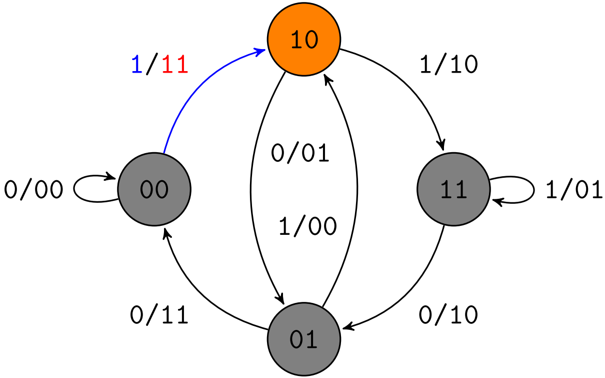
$$\mathbf{v} = 1110101111$$



Encodage

$\mathbf{u} = 1100100$

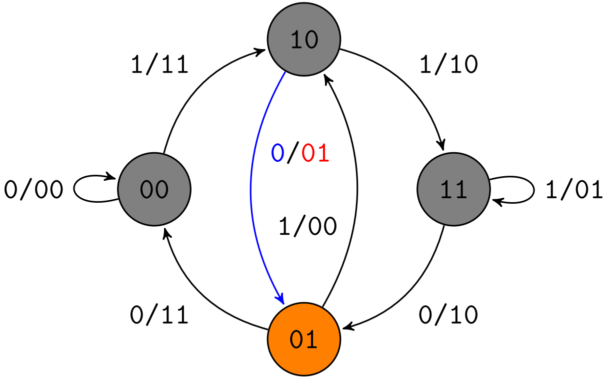
$\mathbf{v} = 11101011 11$



Encodage

$$\mathbf{u} = 1100100$$

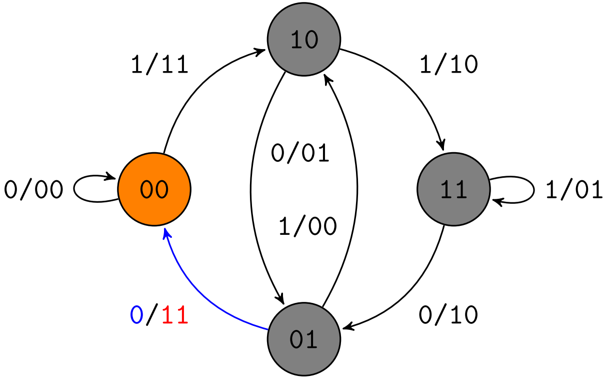
$$\mathbf{v} = 11101011101$$



Encodage

$\mathbf{u} = 1100100$

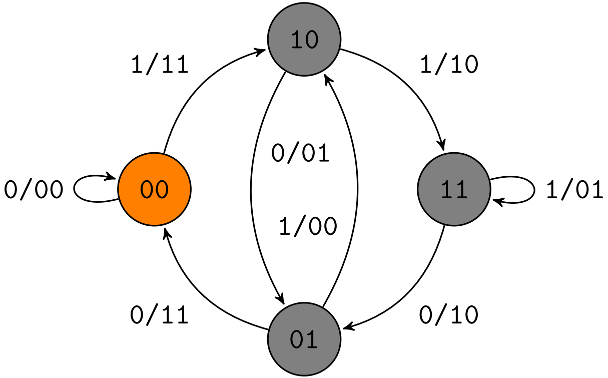
$\mathbf{v} = 11101011110111$



Encodage

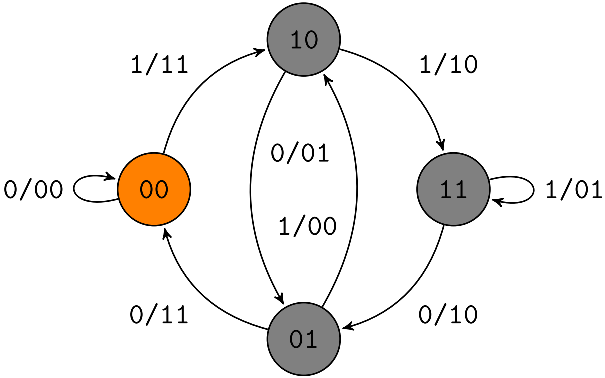
$$\mathbf{u} = 1100100$$

$$\mathbf{v} = 11101011110111$$



$$\mathbf{u} = ?$$

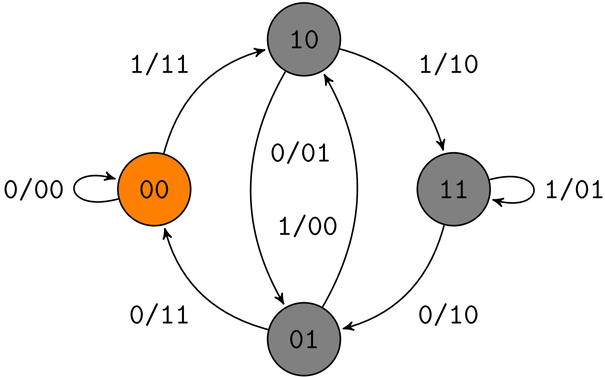
$$\tilde{\mathbf{v}} = 11001111010111$$



Décodage

$$\mathbf{u} = 10111\ ?$$

$$\tilde{\mathbf{v}} = 11001111010111$$

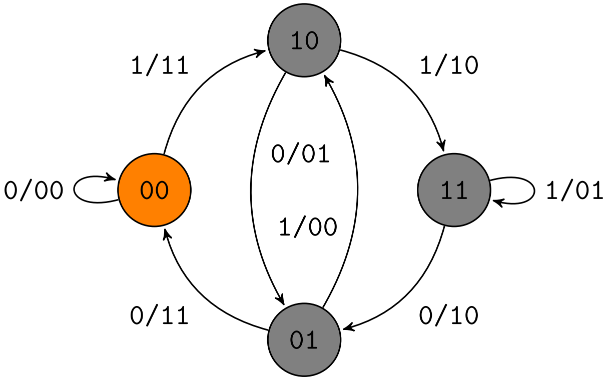


Décodage

$$\mathbf{u} = 10111\ ?$$

$$\mathbf{v} = 11010010011011$$

$$\tilde{\mathbf{v}} = 11001111010111$$

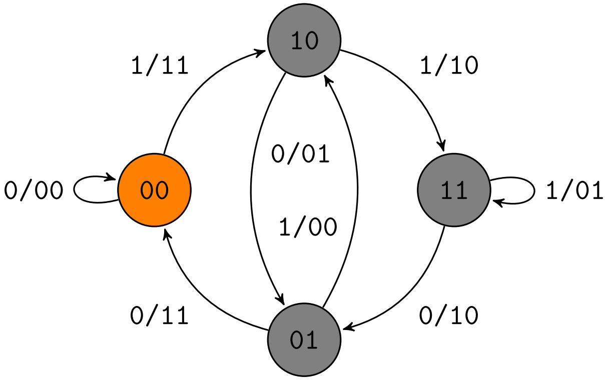


Décodage

$$\mathbf{u} = 10111\ ?$$

$$\mathbf{v} = 11010010011011$$

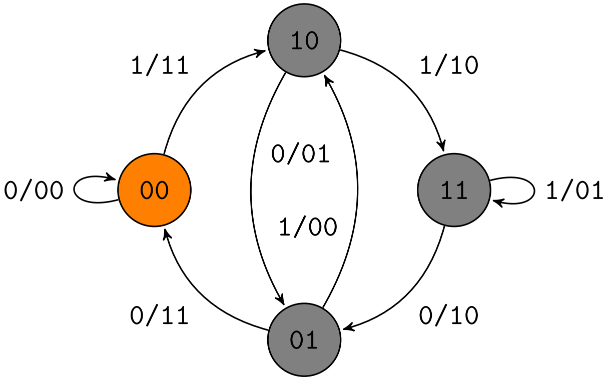
$$\tilde{\mathbf{v}} = 11001111010111$$



$$\mathbf{u} = 10111 \;?\; d = 6$$

$$\mathbf{v} = 11010010011011$$

$$\tilde{\mathbf{v}} = 11001111010111$$



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Il suffit de maintenir pour chaque t la liste des chemins de longueur t de plus faible coût arrivant en chaque état

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- Or il n'y a que m états internes $\implies O(t \cdot 2^m)$

Il suffit de maintenir pour chaque t la liste des chemins de longueur t de plus faible coût arrivant en chaque état (*survivants*)

Survivants à $t = 0$

$\tilde{\mathbf{v}} = 11001111010111$

00 :

10 :

01 :

11 :

Propagation

$\tilde{\mathbf{v}} = 1100111 1010111$

00 : 0

10 : 1

01 :

11 :

Propagation

$\tilde{\mathbf{v}} = 1100111101 0111$

$$
\begin{aligned}
00 &: 0/00 \\
10 &: 1/11 \\
01 &: \\
11 &:
\end{aligned}
$$

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

00 :	0/00	2
10 :	1/11	0
01 :		
11 :		

Survivants à $t = 1$

$\tilde{\mathbf{v}} = 1100111101 0111$

00 : 0	2
10 : 1	0
01 :	
11 :	

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

00 : 00	2
10 : 01	2
01 : 10	0
11 : 11	0

Propagation

$\tilde{\mathbf{v}} = 11\textcolor{blue}{00}1111010111$

$$
\begin{array}{lcr}
00 : 0\textcolor{red}{0}/\textcolor{blue}{00} & \qquad & \textcolor{orange}{2+0} \\
10 : 0\textcolor{red}{1}/\textcolor{blue}{11} & & \textcolor{orange}{2+2} \\
01 : 1\textcolor{red}{0}/\textcolor{blue}{01} & & \textcolor{orange}{0+1} \\
11 : 1\textcolor{red}{1}/\textcolor{blue}{10} & & \textcolor{orange}{0+1}
\end{array}
$$

Survivants à $t = 2$

$\tilde{\mathbf{v}} = 11\color{blue}{00}\color{black}{1111010111}$

00 : 00	2
10 : 01	4
01 : 10	1
11 : 11	1

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

00 :	000	2
	100	1
10 :	001	2
	101	1
01 :	010	4
	110	1
11 :	011	4
	111	1

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

$00 :$	$000/00$	$2+2$
	$100/11$	$1+0$
$10 :$	$001/11$	$2+0$
	$101/00$	$1+2$
$01 :$	$010/01$	$4+1$
	$110/10$	$1+1$
$11 :$	$011/10$	$4+1$
	$111/01$	$1+1$

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

00 :	000	4
	100	1
10 :	001	2
	101	3
01 :	010	5
	110	2
11 :	011	5
	111	2

Survivants à $t = 3$

$\tilde{\mathbf{v}} = 11001111010111$

00 : 100	1
10 : 001	2
01 : 110	2
11 : 111	2

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

00 :	1000	1
	1100	2
10 :	1001	1
	1101	2
01 :	0010	2
	1110	2
11 :	0011	2
	1111	2

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

$$
\begin{array}{lll}
00: & 1000/00 & 1+2 \\
 & 1100/11 & 2+0 \\
10: & 1001/11 & 1+0 \\
 & 1101/00 & 2+2 \\
01: & 0010/01 & 2+1 \\
 & 1110/10 & 2+1 \\
11: & 0011/10 & 2+1 \\
 & 1111/01 & 2+1
\end{array}
$$

Comparaison

$\tilde{\mathbf{v}} = 11001111010111$

00 :	1000	3
	1100	2
10 :	1001	1
	1101	4
01 :	0010	3
	1110	3
11 :	0011	3
	1111	3

Survivants à $t = 4$

$\tilde{\mathbf{v}} = 11001111010111$

00 : 1100	2
10 : 1001	1
01 : 1110	3
11 : 1111	3

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

00 :	11000	2
	11100	3
10 :	11001	2
	11101	3
01 :	10010	1
	11110	3
11 :	10011	1
	11111	3

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

$$
\begin{array}{rlr}
00: & 11000/00 & 2+1 \\
 & 11100/11 & 3+1 \\
10: & 11001/11 & 2+1 \\
 & 11101/00 & 3+1 \\
01: & 10010/01 & 1+0 \\
 & 11110/10 & 3+2 \\
11: & 10011/10 & 1+2 \\
 & 11111/01 & 3+0
\end{array}
$$

Propagation

$\tilde{\mathbf{v}} = 11001111\textcolor{blue}{01}0111$

00 :	11000	3
	11100	4
10 :	11001	3
	11101	4
01 :	10010	1
	11110	5
11 :	10011	3
	11111	3

Survivants à $t = 5$

$\tilde{\mathbf{v}} = 1100111101 0111$

00 : 11000	3
10 : 11001	3
01 : 10010	1
11 : 10011	3

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

00 :	110000	3
	100100	1
10 :		
01 :	110010	3
	100110	3
11 :		

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

$00:$	$110000/00$	$3+1$
	$100100/11$	$1+1$
$10:$		
$01:$	$110010/01$	$3+0$
	$100110/10$	$3+2$
$11:$		

Propagation

$\tilde{\mathbf{v}} = 11001111010111$

00 :	110000	4
	100100	2
10 :		
01 :	110010	3
	100110	5
11 :		

Survivants à $t = 6$

$\tilde{\mathbf{v}} = 110011110\color{blue}{01}\color{black}{11}$

00 :	100100	2
10 :		
01 :	110010	3
11 :		

Propagation

$\tilde{\mathbf{v}} = 1100111101011$

00 :	1001000	2
	1100100	3
10 :		
01 :		
11 :		

Propagation

$\tilde{\mathbf{v}} = 110011110101\textcolor{blue}{11}$

$00 : 100100\textcolor{red}{0}/\textcolor{blue}{00}$ $\qquad$ $\textcolor{orange}{2+2}$

$ 110010\textcolor{red}{0}/\textcolor{blue}{11}$ $\qquad$ $\textcolor{orange}{3+0}$

$10 :$

$01 :$

$11 :$

Propagation

$\tilde{\mathbf{v}} = 1100111101011$

00 :	1001000	4
	1100100	3
10 :		
01 :		
11 :		

Survivant à $t = 7$

$\tilde{\mathbf{v}} = 11001111010111$

00 :	1100100	3
10 :		
01 :		
11 :		

Survivant à $t = 7$

$\tilde{\mathbf{v}} = 11001111010111$

00 :	1100100	3
10 :		
01 :		
11 :		

$\implies$ $\mathbf{u} = 11001$ message le plus vraisemblable